Could It Be Groovy to Be a Square?

12.6

Approximating and Rewriting Radicals

LEARNING GOALS

In this lesson, you will:

- Determine the square root of perfect squares.
- Determine the approximate square root of given values.
- Determine the exact value of a square root of given values.
- Rewrite radicals by extracting perfect squares.

KEY TERMS

- square root
- positive square root
- principal square root
- negative square root
- extract the square root
- radical expression
- radicand

One of the most brilliant of ancient Greek mathematicians was a man named Pythagoras. He believed that every number could be expressed as a ratio of two integers.

Yet, legend tells us that one day at sea, one of Pythagoras's students pointed out to him that the diagonal of a square which measures 1 unit by 1 unit would be $\sqrt{2}$, a number which could not possibly be represented as a ratio of two integers.

This student was allegedly thrown overboard and the rest of the group was sworn to secrecy!

PROBLEM 1 Good Vibrations

Vanessa plays guitar and knows that when she plucks a guitar string, the vibrations result in sound waves, which are the compression and expansion of air particles. Each compression and expansion is called a cycle. The number of cycles that occur in one second is 1 hertz. The number of cycles that occur in one second can be referred to as "wave speed," or "frequency."

Vanessa also knows that tuning a guitar requires changing the tension of the strings. The tension can be thought of as the amount of stretch (in pounds of pressure per inch) on a string between two fixed points. A string with the correct tension produces the correct number of cycles per second over time, which produces the correct tone.

1. Consider a guitar string that has a tension of 0.0026 pound per inch. An equation that relates hertz h and tension t in pounds per inch is $h^2 = t$.
   a. Determine the string tension if the frequency is 9.5 hertz.

   b. Determine the string tension if the frequency is 7.6 hertz.

   c. Determine the string tension if the frequency is 8.5 hertz.

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2. What appears to happen to the tension as the frequency increases?

3. Write an equation to determine the frequency produced by a string with a tension of 81 pounds per inch. Use the same variables you were given in Question 1.

4. Write an equation to determine the frequency produced by a string with a tension of 36 pounds per inch. Use the same variables you were given in Question 1.

Notice that your answers to Questions 3 and 4 are *square roots* of 81 and 36. A number b is a **square root** of a if $b^2 = a$. So, 9 is the square root of 81 because $9^2 = 81$, and 6 is a square root of 36 because $6^2 = 36$.

5. Jasmine claims that 81 could have two square roots: 9 and −9. Maria says that there can be only one square root for 81, which is 9. Determine who is correct and explain why that student is correct.

In earlier grades, you may have seen problems in which you only determined one square root. However, there are in fact two square roots for every whole number, a **positive square root** (which is also called the **principal square root**) and a **negative square root**. This occurs because of the rule you learned about multiplying two negative numbers: when two negative numbers are multiplied together, the product is a positive number.

To solve the equation

$$h^2 = 81,$$

you can **extract the square root** from both sides of the equation.

$$\sqrt{h^2} = \pm\sqrt{81}$$
$$h = \pm 9$$

However, you must still be mindful of the solutions in the context of the problem.

6. Lincoln determined the frequency, in hertz, for a string with a tension of 121 pounds per inch. His work is shown.

Lincoln

$h^2 = 121$
$h = \pm\sqrt{121}$
$h^2 = \pm 11$
The frequency must be 11 hertz because the square of 11 is 121.

Explain why Lincoln is correct in the context of the problem.

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PROBLEM 2 Ah, That's Close Enough . . .

Generally, experts agree that humans can hear frequencies that are between 15 hertz and 20,000 hertz.

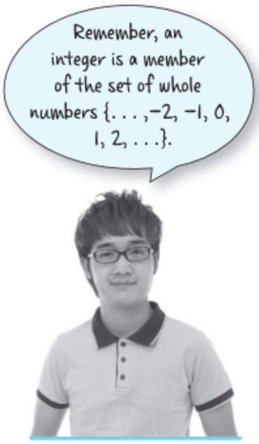
Recall the equation $h^2 = t$. The A-string on a guitar has a frequency of 440 hertz (cycles per second).

1. Determine the string tension if the frequency of the A-string is 440 hertz.

If $440^2 = 193{,}600$, then $\sqrt{193{,}600} = 440$. This second expression is a **radical expression** because it involves a radical symbol ($\sqrt{\ }$). The **radicand** is the expression enclosed within the radical symbol. In the expression $\sqrt{193{,}600}$, 193,600 is the radicand.

2. Write an equation to determine the frequency of a string with a tension of 75 pounds per inch.

3. Write your equation as a radical expression.



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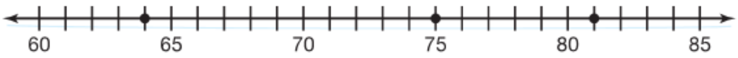
4. Can you predict whether the frequency will be a positive integer? Explain why or why not.

You can also estimate the square roots of numbers that are not perfect squares.

You can determine the approximate value of $\sqrt{75}$.

Determine the perfect square that is closest to but less than 75.
Then determine the perfect square that is closest to but greater than 75.

$64 \le 75 \le 81$



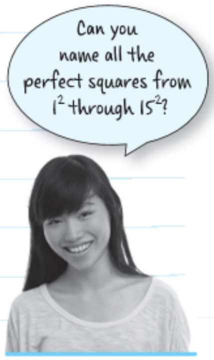
Determine the square roots of the perfect squares.

$\sqrt{64} = 8$ $\sqrt{75} = ?$ $\sqrt{81} = 9$

Now that you know that $\sqrt{75}$ is between 8 and 9, you can test the squares of numbers between 8 and 9.

$8.6^2 = 73.96$ $8.7^2 = 75.69$

Since 75.69 is closer to 75 than 73.96, 8.7 is the approximate square root of $\sqrt{75}$.



5. Determine the approximate frequency for each string tension given. First, write a quadratic equation with the information given, and then approximate your solution to the nearest tenth.

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   a. 42 pounds per inch

   b. 50 pounds per inch

c. 136 pounds per inch

6. The lowest key on a piano has a string tension of 756 pounds of pressure per inch. What is the frequency of the lowest key on the piano? Write an equation and approximate your answer to the nearest tenth.

PROBLEM 3 No! It Must Be Exact

There are times when an exact solution is necessary. For example, acoustical engineers may need to calculate the exact solution when designing sound stages or studios.

Laura makes and designs acoustical tiles for recording studios. These tiles are used to reflect different instrument tones of different frequencies in the studio. One of her clients has requested that the area of each square acoustical tile needs to be 75 square inches. Because these tiles can be lined up in rows or columns and they affect other factors in a recording studio, Laura needs to determine the exact side measure of each acoustical tile.

Consider a square acoustical tile with an area of 75 square inches.

You can set up and solve an equation to determine exact side length of the square.

$$s^2 = 75$$

$$s = \sqrt{75}$$

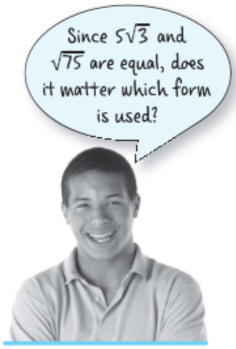
You can also rewrite $\sqrt{75}$ in a different form to help show the value. First, rewrite the product of 75 to include any perfect square factors, and then extract the square roots of those perfect squares.

$$\begin{aligned}\sqrt{75} &= \sqrt{25 \cdot 3} \\ &= \sqrt{25} \cdot \sqrt{3} \\ &= \sqrt{5 \cdot 5} \cdot \sqrt{3} \\ &= 5\sqrt{3}\end{aligned}$$

The exact measure of each side of the acoustical tile is $\sqrt{75}$, or $5\sqrt{3}$ inches.

1. Estimate the value of $5\sqrt{3}$. Explain your reasoning.

2. Compare your approximation of $5\sqrt{3}$ to the approximation of $\sqrt{75}$ from the worked example in Problem 2. What do you notice?



3. Rewrite each radical by extracting all perfect squares, if possible.

a. $\sqrt{20}$

b. $\sqrt{26}$

c. $\sqrt{64}$

4. For each given area, write an equation to determine the side measurements of the square acoustical tiles. Then, determine the exact side measurement of each square acoustical tile.

a. 18 square inches

b. 116 square inches

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Can extracting the square root also be used for expressions containing a variable and a constant term that are squared?

Consider the equation $(x - 1)^2 = 17$. How can you determine the value of x?

First take the square root of both sides of the equation.

$$(x - 1)^2 = 17$$

$$\sqrt{(x - 1)^2} = \pm\sqrt{17}$$

$$x - 1 = \pm\sqrt{17}$$

Then, isolate the variable to determine the value of the variable.

$$x - 1 + 1 = \pm\sqrt{17} + 1$$

$$x = 1 \pm \sqrt{17}$$

$$x \approx 1 \pm 4.12$$

$$x \approx -3.12 \quad \text{or} \quad 5.12$$

5. Determine the approximate solutions for each of the given equations.

a. $(r + 8)^2 = 83$ **b.** $(17 - d)^2 = 55$

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6. Rewrite each radical by extracting all perfect squares, if possible.

a. $\sqrt{18}$

b. $\sqrt{116}$

c. $5\sqrt{24}$

d. $7\sqrt{99}$

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Be prepared to share your solutions and methods.